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EQUATIONS AND PROCEDURES FOR NUMERICALLY CALCULATING THE AERODYNAMIC CHARACTERISTICS OF LIFTING ROTORS

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SUMMARY

Equations and procedures are presented for carrying out the numerical operations involved in computing the aerodynamic characteristics of lifting rotors by means of modern high-speed automatic computers. The characteristics considered herein include the thrust, profile drag, total power, flapping, rolling and pitching moments, direction of the resultant-force vector, and the harmonic contribution of the shear-force input to the hub. The equations are general and can account for stall and compressibility effects, hinge offset, and other factors that are normally omitted from conventional analytical rotor treatments.

INTRODUCTION

The problem of determining the aerodynamic characteristics of a lifting rotor basically consists of calculating the forces and moments on a rotating three-dimensional lifting surface from statically measured two-dimensional airfoil-section data. Because of the large number of degrees of freedom involved and the complications introduced by the variation in section angles of attack and velocities over the rotor disk, an analytical approach to the problem necessarily contains various assumptions and simplifications. The largest number of such simplifying assumptions is found in the original efforts of Glauert and Lock, published almost 30 years ago. Since that time, the theory has been progressively improved in accuracy and convenience of application by a number of investigators, and has been extended to meet the needs of new designs and more severe flight conditions.

These extensions and improvements in accuracy, however, have been realized at the expense of a very large increase in the length and complexity of the resulting equations when closed-form solutions are desired. (See ref. 1, for example.) In order to make the application of these equations practical, it is usually necessary to evaluate the equations over a range of flight parameters and to present the results in chart form. (For example, see refs. 2 to 4.) Such solutions, in spite of their complexity, still do not account for blade stall outside of the reversed-velocity region, compressibility effects, and combinations of such design factors as hinge offset, blade twist and taper, and root cutout.

Instead of trying to include such factors in the already complicated equations by first integrating the general equations and then substituting particular values, it would seem more practical to account for these factors numerically; that is, by substituting numerical values into the differential rotor equations and integrating numerically. This procedure is made practicable only by the development in the last few years of high-speed automatic computing machines, inasmuch as the numerical work involved in evaluating a large number of cases by manual calculating methods is prohibitive. The availability of such computing machines to industry as well as to research institutions, through ownership or rental, makes the numerical procedures appropriate for design as well as for research studies.

It is the purpose of this paper to present the equations and procedures necessary for numerically computing the aerodynamic characteristics of lifting rotors. The characteristics considered herein include the thrust, profile-drag power, total power, flapping, rolling and pitching moments, direction of the resultant-force vector (necessary in stability calculations), and the harmonic contribution of each blade of the rotor to the shear-force input to the hub.

SYMBOLS

A_1, B_1 coefficients of $-\cos\psi$ and $-\sin\psi$, respectively, in expression for θ; therefore, lateral and longitudinal cyclic pitch, respectively, deg

a speed of sound, ft/sec

a' projection of angle between rotor resultant-force vector and shaft axis in plane containing flight path and shaft axis, $\tan^{-1}\frac{C_H}{C_T}$

a_0 constant term in Fourier series that expresses β, radians; hence, rotor coning angle

a_n coefficient of $\cos n\psi$ in expression for β

B tip-loss factor; blade elements outboard of radius BR are assumed to have profile drag but no lift

b	number of blades per rotor
b'	projection of angle between rotor resultant-force vector and shaft axis in plane perpendicular to plane containing flight path and shaft axis, $\tan^{-1} \frac{C_Y}{C_T}$
b_n	coefficient of $\sin n\psi$ in expression for β
C_O	constant term in Fourier series that expresses $M_T/I_h\Omega^2$
C_H	rotor longitudinal-force coefficient, $\frac{H}{\pi R^2 \rho (\Omega R)^2}$
C_l	rolling-moment coefficient, $\frac{L'}{\pi R^2 \rho (\Omega R)^2 R}$
C_m	pitching-moment coefficient, $\frac{M}{\pi R^2 \rho (\Omega R)^2 R}$
C_n	coefficient of $\cos n\psi$ in expression for $M_T/I_h\Omega^2$
C_P	rotor-shaft power coefficient, $\frac{P}{\pi R^2 \rho (\Omega R)^3}$
C_Q	rotor-shaft torque coefficient, $\frac{Q}{\pi R^2 \rho (\Omega R)^2 R}$
C_T	rotor thrust coefficient, $\frac{T}{\pi R^2 \rho (\Omega R)^2}$
$C_T(\psi)$	C_T value at a given azimuth position ψ, expressed by series $C_T(\psi) = E_O + E_1 \cos \psi + F_1 \sin \psi + E_2 \cos 2\psi + F_2 \sin 2\psi + E_3 \cos 3\psi + F_3 \sin 3\psi$
C_V	shear-force coefficient at flapping hinge due to mass forces, $\frac{F_m}{\pi R^2 \rho (\Omega R)^2}$
C_Y	rotor lateral-force coefficient, $\frac{Y}{\pi R^2 \rho (\Omega R)^2}$
c	blade-section chord at radial station x, ft

$c_{d,o}$ section profile-drag coefficient

c_e equivalent blade chord (on thrust basis), $\dfrac{\int_{r_c}^{BR} cr^2\,dr}{\int_{r_c}^{BR} r^2\,dr}$, ft

c_e' equivalent blade chord (on thrust-moment basis), $\dfrac{\int_{r_c}^{BR} cr^3\,dr}{\int_{r_c}^{BR} r^3\,dr}$, ft

c_l section lift coefficient

c_t blade chord at tip, ft

D_n coefficient of $\sin n\psi$ in expression for $M_T/I_h\Omega^2$

E_0 constant term in series expression for $C_T(\psi)$; $E_0 = C_T$

E_n coefficient of $\cos n\psi$ in expression for $C_T(\psi)$

e offset of center line of flapping hinge from center line of rotor shaft, ft

F_m shear force at flapping hinge (perpendicular to blade-span axis) due to mass forces, lb

F_n coefficient of $\sin n\psi$ in expression for $C_T(\psi)$

g gravitational acceleration, ft/sec^2

H component of rotor resultant force perpendicular to rotor shaft in longitudinal plane of symmetry, lb

I_h mass moment of inertia of blade about flapping hinge, $\int_e^R m(r - e)^2\,dr$, slug-ft^2

$k = \dfrac{em_b}{M_{m,s}}$

L' rotor rolling moment, lb-ft

M	rotor pitching moment, lb-ft
$M_{m,s}$	mass moment of blade about center line of rotor shaft, slug-ft
M_T	thrust moment of blade about flapping hinge, lb-ft
M_W	weight moment of blade about flapping hinge, $\int_e^R mg(r - e)dr$, lb-ft
M_x	Mach number of blade section, $u\Omega R/a$
m	mass of blade per foot of radius, slugs/ft
m_b	mass of blade, slugs
P	shaft power, ft-lb/sec
p	helicopter rolling velocity, radians/sec
Q	shaft torque, lb-ft
q	helicopter pitching velocity, radians/sec
R	blade tip radius measured from center of rotation, ft
r	distance measured along blade from axis of rotation to blade element, ft
T	component of rotor resultant force acting along rotor shaft, lb
u	nondimensional resultant velocity perpendicular to blade-span axis at blade element
u_P	component at blade element of u parallel to shaft axis
u_T	component at blade element of u perpendicular to blade-span axis and to shaft axis
V	true airspeed of helicopter along flight path, ft/sec
v	induced inflow velocity at rotor, ft/sec
x	ratio of blade-element radius to rotor-blade radius, r/R
Y	lateral component of rotor resultant force perpendicular to both T and H, lb
α	angle between shaft axis and plane perpendicular to flight path, positive when axis is pointing rearward, deg

α_r blade-element angle of attack, measured from line of zero lift, deg

β blade flapping angle with respect to shaft at a particular azimuth position, $a_0 - a_1 \cos \psi - b_1 \sin \psi - a_2 \cos 2\psi \ldots$, radians

γ' mass constant of blade, $\frac{\rho c_e' R^4}{I_h}$

γ'_x local value of blade mass constant, $\frac{\rho c R^4}{I_h}$

$\eta = \frac{e}{g} \frac{M_W}{I_h}$

θ_0 collective-pitch angle at blade root, average value of instantaneous blade-root pitch angle around azimuth, deg

θ_1 difference between blade-root and blade-tip pitch angles, positive when tip angle is larger, deg

$\theta_{.75}$ blade-section pitch angle at 0.75 radius, deg

Θ instantaneous blade-section pitch angle; angle between line of zero lift of blade section and plane perpendicular to rotor shaft, $\theta_0 + \theta_1 x - A_1 \cos \psi - B_1 \sin \psi$, deg

λ inflow ratio, $\frac{V \sin \alpha - v}{\Omega R}$

μ tip-speed ratio, $\frac{V \cos \alpha}{\Omega R}$

$\xi = e/R$

ρ mass density of air, slugs/cu ft

σ rotor solidity, $bc_e/\pi R$

σ_x local solidity, $bc/\pi R$

ϕ inflow angle at blade element in plane perpendicular to blade-span axis, $\tan^{-1} \frac{u_P}{u_T}$, deg

ψ blade azimuth angle measured from downwind position in direction of rotation, deg

Ω rotor angular velocity, radians/sec

Subscripts:

c radius of cutout; that is, radius at which lifting surface of blade begins

a accelerating

d decelerating

o profile

i induced

l lift

METHOD OF ANALYSIS

The process of numerically determining the aerodynamic characteristics of a rotor operating at a particular flight condition consists of calculating the individual force (and moment, if desired) contributions of a specific number of blade sections at various points on the rotor disk, averaging the values around the disk at a particular radial station, and then radially integrating these averages along the blade to obtain the result. The equations required to evaluate the force contribution of a blade section and the procedure for utilizing the equations are written specifically for a hinged (i.e., free-to-cone) rotor, but can be applied with but little modification to seesaw rotors. The modification of the equations to the seesaw-rotor configuration is discussed in reference 5.

Reference-Axis System

In the analysis developed herein, the flapping hinge may be offset radially from the center line of the shaft. For such a configuration, flapping and feathering motions are not directly equivalent and it is therefore most convenient to reference all angles and velocities to an axis coinciding with the center line of the rotor shaft. The rotor shaft is therefore used as the reference axis in this paper instead of the no-feathering axis which was used in references 1 to 4.

Equations

Most of the equations listed in this section are based on, or can be readily derived from, standard blade-element equations. (See refs. 1 and 6, for example.) Because the equations are set up for numerical integration, most of the aerodynamic restrictions that are normally employed in rotor analyses are avoided. Thus, the equations can be used for blades of any airfoil section, mass distribution, twist, plan-form taper, root cutout, and flapping-hinge offset and can account for the effects of helicopter pitching and rolling velocities. Flapping-hinge configurations wherein blade flapping and lagging motions result in pitch-angle changes can be simply considered by the addition of the appropriate terms in the equation for section angle of attack presented in this section. Stall and compressibility effects can be accounted for to the extent of using actual airfoil data that correspond to the range of angle of attack and Mach number encountered at all points of the disk, including the reversed-velocity region. Although no small-angle limitations have been made regarding the section inflow angles or the treatment of the magnitude of the resultant velocities and the direction of the resultant-force vectors at the blade sections, the analysis does incorporate small-angle assumptions in regards to the blade flapping angles. In addition, it is assumed that the rotor blades are rigid in bending and torsion and that radial-flow effects can be neglected. The equations can also be applied to any known or assumed radial or azimuth variation of induced velocity.

Input quantities.- The input parameters fixing the flight condition are λ, $\theta_{.75}$, and μ. These parameters may be either assumed directly as independent variables or estimated from known parameters (such as thrust and shaft power) by means of published rotor-performance charts (such as those of refs. 2 and 3). In addition to the flight parameters, the following design and operating variables are assumed to be known: θ_1, A_1, B_1, $c(x)$, I_h, e, M_W, Ω, b, R, ρ, p, q, and $m(x)$, as well as two-dimensional blade-section airfoil data.

Equations for the calculation of section velocities, angles of attack, and Mach number.- Equations that are necessary for the calculation of section velocities, angles of attack, and Mach number are listed as follows:

$$u_T = x + \mu \sin \psi \tag{1}$$

$$\begin{aligned} u_P = {} & \lambda - \left[(x - \xi)a_1 - \frac{p}{\Omega} x\right]\sin \psi + \left[(x - \xi)b_1 + \frac{q}{\Omega} x\right]\cos \psi - \\ & (x - \xi)\left(2a_2 \sin 2\psi - 2b_2 \cos 2\psi + 3a_3 \sin 3\psi - 3b_3 \cos 3\psi\right) - \\ & \mu \cos \psi\left(a_0 - a_1 \cos \psi - b_1 \sin \psi - a_2 \cos 2\psi - b_2 \sin 2\psi - \right. \\ & \left. a_3 \cos 3\psi - b_3 \sin 3\psi\right) \end{aligned} \tag{2}$$

$$u^2 = u_T^2 + u_P^2 \tag{3}$$

$$\phi = \tan^{-1} \frac{u_P}{u_T} \tag{4}$$

$$\begin{aligned} \alpha_r &= \Theta + \phi \\ &= \theta_0 + \theta_1 x - A_1 \cos \psi - B_1 \sin \psi + \phi \end{aligned} \tag{5}$$

$$M_x = \frac{u\Omega R}{a} \tag{6}$$

Equations for the calculation of flapping coefficients.- The flapping coefficients may be calculated from the following relations:

$$\frac{M_T}{I_h\Omega^2} = \frac{1}{2}\left[\int_{x_c}^{B} \gamma'_x u^2(x - \xi)c_l \cos \phi \, dx + \int_{x_c}^{1.0} \gamma'_x u^2(x - \xi)c_{d,o} \sin \phi \, dx\right] \tag{7}$$

$$\frac{M_T}{I_h\Omega^2} = C_0 + C_1 \cos \psi + D_1 \sin \psi + C_2 \cos 2\psi + D_2 \sin 2\psi + C_3 \cos 3\psi + D_3 \sin 3\psi \tag{8}$$

$$a_0 = \left(C_0 - \frac{M_W}{I_h\Omega^2}\right)\frac{1}{1 + \eta} \tag{9}$$

$$\Delta a_1 = \frac{D_1 + \left[\eta\left(b_1 + \Delta b_1\right) - 2 \frac{q}{\Omega}(1 + \eta g)\right]}{\frac{B^2}{8}\left(B^2 - \frac{1}{2} \mu^2\right)\gamma' a} \qquad (\mu \leqq 1) \tag{10a}$$

$$\Delta a_1 = \frac{D_1 + \left[\eta\left(b_1 + \Delta b_1\right) - 2 \frac{q}{\Omega}(1 + \eta g)\right]}{\frac{B^4}{8} \gamma' a} \qquad (\mu > 1) \tag{10b}$$

$$\Delta b_1 = \frac{C_1 + \left[\eta\left(a_1 + \Delta a_1\right) + 2\,\frac{p}{\Omega}(1 + \eta g)\right]}{-\frac{B^2}{8}\left(B^2 + \frac{1}{2}\,\mu^2\right)\gamma' a} \tag{11}$$

$$a_2 = \frac{C_2}{3 - \eta} \tag{12}$$

$$b_2 = \frac{D_2}{3 - \eta} \tag{13}$$

$$a_3 = \frac{C_3}{8 - \eta} \tag{14}$$

$$b_3 = \frac{D_3}{8 - \eta} \tag{15}$$

Equations for the calculation of performance parameters.- The performance parameters may be determined from the following equations:

$$\left(\frac{dC_T}{dx}\right)_l = \frac{1}{2}\,\sigma_x u^2 c_l \cos\phi \tag{16}$$

$$\left(\frac{dC_T}{dx}\right)_o = \frac{1}{2}\,\sigma_x u^2 c_{d,o} \sin\phi \tag{17}$$

$$C_{T,l} = \int_{x_c}^{B} \frac{1}{n} \sum_{\psi=1}^{\psi=n} \left(\frac{dC_T}{dx}\right)_l dx \tag{18}$$

$$C_{T,o} = \int_{x_c}^{1.0} \frac{1}{n} \sum_{\psi=1}^{\psi=n} \left(\frac{dC_T}{dx}\right)_o dx \tag{19}$$

where $\sum_{\psi=1}^{\psi=n}$ refers to the summation of $\frac{dC_T}{dx}$ values computed at n azimuth stations. $\left(\text{For example, } \psi = 0^\circ, \frac{360^\circ}{n}, \frac{2(360^\circ)}{n}, \ldots\right)$

Also

$$C_T = C_{T,l} + C_{T,o} \tag{20}$$

$$\frac{dC_{Q,a}}{dx} = \frac{1}{2}\sigma_x u^2 x c_l \sin \phi \tag{21}$$

$$\frac{dC_{Q,d}}{dx} = \frac{1}{2}\sigma_x u^2 x c_{d,o} \cos \phi \tag{22}$$

$$C_{Q,a} = \int_{x_c}^{B} \frac{1}{n}\sum_{\psi=1}^{\psi=n} \frac{dC_{Q,a}}{dx}\,dx \tag{23}$$

$$C_{Q,d} = \int_{x_c}^{1.0} \frac{1}{n}\sum_{\psi=1}^{\psi=n} \frac{dC_{Q,d}}{dx}\,dx \tag{24}$$

$$C_Q = C_{Q,d} - C_{Q,a} \tag{25}$$

$$\frac{dC_{P,o}}{dx} = \frac{1}{2}\sigma_x u^3 c_{d,o} \tag{26}$$

$$C_{P,o} = \int_{x_c}^{1.0} \frac{1}{n}\sum_{\psi=1}^{\psi=n} \frac{dC_{P,o}}{dx}\,dx \tag{27}$$

Equations for the calculation of rotor forces and moments.- The following equations may be used to calculate certain rotor force and moment coefficients that may be useful in vibration and stability analyses:

$$dC_{H,i} = \frac{1}{2}\sigma_x u^2 c_l(-\sin \phi \sin \psi - \cos \phi \sin \beta \cos \psi)dx \tag{28}$$

$$dC_{H,o} = \frac{1}{2}\sigma_x u^2 c_{d,o}(\cos \phi \sin \psi - \sin \phi \sin \beta \cos \psi)dx \tag{29}$$

$$C_{H,i} = \int_{x_c}^{B} \frac{1}{n}\sum_{\psi=1}^{\psi=n} \frac{dC_{H,i}}{dx}\,dx \tag{30}$$

$$C_{H,o} = \int_{x_c}^{1.0} \frac{1}{n} \sum_{\psi=1}^{\psi=n} \frac{dC_{H,o}}{dx}\, dx \tag{31}$$

$$C_H = C_{H,i} + C_{H,o} \tag{32}$$

$$dC_{Y,i} = \frac{1}{2} \sigma_x u^2 c_l (\sin \phi \cos \psi - \cos \phi \sin \beta \sin \psi) dx \tag{33}$$

$$dC_{Y,o} = \frac{1}{2} \sigma_x u^2 c_{d,o} (-\cos \phi \cos \psi - \sin \phi \sin \beta \sin \psi) dx \tag{34}$$

$$C_{Y,i} = \int_{x_c}^{B} \frac{1}{n} \sum_{\psi=1}^{\psi=n} \frac{dC_{Y,i}}{dx}\, dx \tag{35}$$

$$C_{Y,o} = \int_{x_c}^{1.0} \frac{1}{n} \sum_{\psi=1}^{\psi=n} \frac{dC_{Y,o}}{dx}\, dx \tag{36}$$

$$C_Y = C_{Y,i} + C_{Y,o} \tag{37}$$

$$a' = \tan^{-1} \frac{C_H}{C_T} \tag{38}$$

$$b' = \tan^{-1} \frac{C_Y}{C_T} \tag{39}$$

$$C_T(\psi) = \frac{1}{2}\left(\int_{x_c}^{B} \sigma_x u^2 c_l \cos \phi \, dx + \int_{x_c}^{1.0} \sigma_x u^2 c_{d,o} \sin \phi \, dx\right) \tag{40}$$

After $C_T(\psi)$ is evaluated at different azimuth stations, a harmonic analysis would express $C_T(\psi)$ as a function of ψ as follows:

$$C_T(\psi) = E_0 + E_1 \cos \psi + F_1 \sin \psi + E_2 \cos 2\psi + F_2 \sin 2\psi + E_3 \cos 3\psi + F_3 \sin 3\psi \tag{41}$$

Then

$$\frac{C_l}{\sigma} = -\frac{\xi F_1}{2\sigma} \tag{42}$$

$$\frac{C_m}{\sigma} = -\frac{\xi E_1}{2\sigma} \tag{43}$$

Equations (42) and (43) represent aerodynamic rolling and pitching moments. Blades with offset flapping hinges also produce hub moments due to centrifugal forces as given by the following equations:

$$\Delta \frac{C_l}{\sigma} = b_1 \frac{\xi}{\gamma'} \frac{1 + \xi}{1 - \xi} \tag{44}$$

$$\Delta \frac{C_m}{\sigma} = a_1 \frac{\xi}{\gamma'} \frac{1 + \xi}{1 - \xi} \tag{45}$$

The oscillating shear force at the flapping hinge (perpendicular to the blade-span axis) is composed of thrust and mass forces. The thrust-force contribution, expressed in harmonic form, is given by equation (41). The nondimensional mass-force contribution, made up of centrifugal and inertia forces, is represented by

$$C_V = -\frac{bM_{m,s}}{\pi R^4 \rho} \Big[\big(a_0 - a_1 \cos \psi - b_1 \sin \psi - a_2 \cos 2\psi - b_2 \sin 2\psi - a_3 \cos 3\psi - b_3 \sin 3\psi\big) + (1 - k)\big(a_1 \cos \psi + b_1 \sin \psi + 4a_2 \cos 2\psi + 4b_2 \sin 2\psi + 9a_3 \cos 3\psi + 9b_3 \sin 3\psi\big)\Big] \tag{46}$$

Inasmuch as equations (41) and (46) contain no aerodynamic damping due to bending or bending inertia terms, they apply only to inflexible blades.

Calculation Procedure

The procedure for calculating rotor characteristics is outlined as follows:

(1) For the given input quantities, assume or calculate an approximate set of flapping coefficients. (The charts of ref. 4 may be used to calculate these approximate coefficients if a_3 and b_3 are assumed to be equal to zero.)

(2) With the given input quantities and the approximate flapping coefficients, calculate section velocities and angles of attack by means of equations (1) to (5).

(3) Calculate section Mach numbers by means of equation (6).

(4) By using the calculated angles of attack and Mach numbers, obtain the corresponding values of c_l and $c_{d,o}$ from appropriate two-dimensional section data.

(5) Calculate $M_T/I_h\Omega^2$ by means of equation (7), make a harmonic analysis of the result, and express the answer in the form of equation (8).

(6) Calculate new values of flapping coefficients by means of equations (9) to (15). (The incremental first-harmonic coefficients given by eqs. (10) and (11) should be added as correction factors to the output values from the preceding iteration.)

(7) Repeat steps (2) to (6) until the output values of flapping coefficients differ negligibly from the input values.

(8) Compute β for each value of ψ, using the final values of flapping coefficients.

(9) With the use of the lift and drag coefficients and velocities corresponding to the final set of flapping coefficients, calculate the performance and stability parameters by means of equations (16) to (45).

It will be noted that additional performance, stability, and loads parameters can be calculated with little additional effort, inasmuch as the section velocities, angles of attack, and lift and drag coefficients have already been computed.

DISCUSSION OF METHOD

Assumptions of Theory

Although the numerical integration procedure eliminates many of the restrictive assumptions and simplifications of conventional rotor theory, a number of assumptions still remain. The degree to which these suppositions can affect the validity of the results depends on the specific applications of the method. Perhaps the most important assumption, and one which is inherent in all the theoretical rotor treatments developed thus far, is the use of steady, two-dimensional airfoil-section characteristics in evaluating rotor forces and moments. Although such an approach has been proven successful in predicting rotor characteristics under normal operating conditions, a complete evaluation of its accuracy in conditions involving large amounts of stall and compressibility effects must await further experimental checks. Limited theory-data comparisons that have been made thus far for extreme operating conditions, however, do show good agreement. The application of such theory to these conditions is therefore felt to be the best step which can be taken for preliminary estimates at the present time. As more experimental data become available, comparison with these predictions will serve to show whether or not empirical or other revised approaches are necessary.

The assumption of blade inflexibility in bending and torsion would normally be unimportant except in aeroelastic problems. Bending flexibility should be considered, for example, in the computation of coupled blade-fuselage vibrations, whereas torsion flexibility would enter into blade-fuselage clearance calculations or in stability or stress calculations wherein a large blade-section center-of-pressure shift associated with operation at high Mach numbers occurs. The neglect of radial-velocity components along the blade would probably be significant only at very high tip-speed ratios wherein they might influence the profile-drag power and the stall pattern of the rotor.

An important limitation inherent in the analysis, and one whose effects can be readily evaluated, is the use of small-angle assumptions as applied to the rotor-blade flapping angle. The flapping analysis containing these assumptions yields erroneous answers if the amplitude of the flapping angle is in the neighborhood of 15° or above. For flight conditions involving larger amplitudes, the numerical method for calculating blade flapping presented in reference 5, which contains no small-angle limitations, should be used. (The two methods yield identical answers for conditions wherein the flapping angles are below approximately 15°.) The method of reference 5 can also be used for cases in which a knowledge of the transient blade-flapping behavior in maneuvers is desired.

Application of Method

Aside from the assumptions just discussed, the accuracy of the answers obtained from the method of this paper is dependent upon the number of radial and azimuth stations around the disk used in the integral force, moment, and power equations. The problem is to determine the point of diminishing returns at which further increases in accuracy obtained through the use of a greater number of stations are counterbalanced by the additional time and cost required to compute the extra stations. The answer to this problem is a variable and is dependent upon the flight condition for which the answer is to be calculated (i.e., the extent of the regions on the disk undergoing stall, compressibility, and reversed-velocity effects), the type of answer to be calculated (for example, thrust loading for bending-moment calculations, stability derivatives, or blade flapping), and the type of automatic computing equipment available for the calculations.

For example, because the calculation of the flapping coefficients, which is necessarily an iterative procedure, is quite lengthy, it is important to determine the initial estimate as closely as possible and to require no more accuracy than is necessary for the particular application. Thus, for cases in which the determination of the flapping coefficients is not in itself the objective but is merely the means for calculating such items as rotor thrust or power, it is usually sufficiently accurate to ignore the third, and perhaps even the second, harmonics of flapping, and to keep the number of iterations at a minimum. Such considerations become less important, of course, when the calculations are carried out on a modern high-speed automatic computer. Conversely, if the flapping motion is desired in its own right, the number of iterations (and the number of harmonics) can be carried out to any desired accuracy.

In order to obtain an idea of the effect of the number of stations on the accuracy of the computations, rotor flapping coefficients (called part I solutions) and thrust, power, and direction of the resultant-force vectors (called part II solutions) were evaluated for a sample flight condition by using different numbers of radial and azimuth stations. Angle-of-attack and Mach number contours corresponding to the sample case are plotted in figure 1 to illustrate the rate of change of these quantities with radius and azimuth angle. As can be seen from the figure, the flight condition chosen was rather extreme and involved large amounts of stall and compressibility effects at a tip-speed ratio of 0.50 and a tip speed equal to 750 feet per second. The results of the calculations are presented in table I.

The results shown in table I were obtained at thirty-six ($\Delta\psi = 10^{\circ}$), eighteen ($\Delta\psi = 20^{\circ}$), and eight ($\Delta\psi = 45^{\circ}$) azimuth stations. In the radial direction, values were computed at $x = 0.15$ and at $x = 1.0$, with seven ($\Delta x = 0.1063$) and three ($\Delta x = 0.2125$) uniformly spaced stations in between. (For those quantities, such as thrust and induced torque, which require the integrations to be carried out to $x = 0.97$ instead of $x = 1.0$, values were also computed at $x = 0.97$. The value of the quantity between $x = 0.97$ and $x = 1.0$, computed by the trapezoidal rule, was subtracted from the total value of the quantity obtained by integrating from $x = 0.15$ to $x = 1.0$.) With the exception noted in the table, all the part II solutions were computed with the same set of flapping coefficients (those corresponding to $\Delta x = 0.2125$, $\Delta\psi = 45^{\circ}$) in order to isolate the effect of changes in the number of stations on the results.

It can be seen that, even for the extreme flight condition considered, the differences in flapping angles resulting from the use of a different number of stations are small, being of the order of a few tenths of a degree for the coning angle and first harmonics, and even less for the second and third harmonics. (For these cases, the number of iterations was such that the values listed in the table differed from those of the preceding iteration by less than 0.06° for a_0, a_1, and b_1 and less than 0.03° for a_2, b_2, and b_3.)

Table I shows that the part II solutions are somewhat more sensitive to the number of stations used than are the flapping coefficients, although not unduly so. By using the results for the case wherein $\Delta x = 0.1063$ and $\Delta\psi = 10^\circ$ as a base, it can be seen that approximately doubling the interval of radial and azimuth stations results in an error of only 1 percent or less in the thrust and power values. When $\Delta\psi$ is increased from 20° to 45°, the error jumps to the order of 5 percent, which, in many cases (such as those involving extensive stall and compressibility effects), is within the order of accuracy and use of the airfoil-section data. In certain calculations, however, such as comparative studies, the introduction of a 5-percent error might be undesirable and could be avoided by doubling the number of azimuth stations. In general, it appears that greater accuracy is achieved with a given increase in stations by increasing the number of azimuth, rather than radial, stations.

In order to illustrate the effect of differences in flapping coefficients on the part II solutions, the case for $\Delta x = 0.2125$ and $\Delta\psi = 45^\circ$ was repeated, but with the second and third flapping coefficients assumed equal to zero. As can be seen in table I, there is a slight increase (about 2 percent) in thrust, and less than one-half of 1 percent decrease in power.

In summary, it appears that if accuracy is a prime consideration, or if a very-high-speed computer (such as an IBM type 704 electronic data processing machine or a Remington Rand UNIVAC) is available, then approximately five radial and eighteen azimuth stations should be used for most part II solutions, whereas the part I (flapping) solutions can be determined with the same accuracy by using five radial and only eight azimuth stations. For slower speed computers, such as the IBM Card-Programmed Electronic Calculator, five radial and eight azimuth stations can be used for the part II solutions with a considerable saving in computing time at the expense of a small reduction in accuracy.

For a given number of stations, the possibility exists that the accuracy of a set of calculations can be improved by nonuniform spacing of the radial and azimuth stations. The possibility is greatest in regards to the radial spacing, particularly when a small number of stations, such as five, is utilized. The effectiveness of a specific choice of the number and spacing of radial stations is also dependent on the manner in which the radial integrations are carried out. Simpson's rule was used in the sample calculations presented in table I and was found to be satisfactory for the type of thrust and power loading curves encountered.

The technical procedures involved in setting up calculations of the type presented in this paper are described in a number of references (see ref. 7, for example) and will not be discussed herein. It is considered worthwhile to point out, however, that approximately 160 hours and 120 hours of programming (i.e., problem setup) time are required for the part I and II solutions, respectively. This part of the work is done only once, of course, and need not be repeated for each new set of input variables. Approximately 18,000 to 20,000 operations are involved in the part I solutions for the 48-station case when compressible section data are included, and approximately an equal number for the part II solutions. The corresponding time required to run off a part I iteration or a part II case for each new set of variables would vary from approximately 100 minutes on an IBM CPC to approximately 20 seconds on an IBM type 704 calculator. The machine time required per case varies almost directly with the number of stations used.

CONCLUDING REMARKS

A numerical method suitable for application to automatic computing machines has been presented for calculating the aerodynamic characteristics of rotors for flight and design conditions that are outside the range of conventional analytical rotor theory. By means of the equations presented, the effects of such items as stall, compressibility, flapping-hinge offset, blade mass factor, twist, taper, and root cutout on such rotor characteristics as flapping, thrust, power, and hub moments, as well as on certain stability derivatives, can be investigated.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., June 6, 1956.

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TABLE I.- EFFECT OF NUMBER OF RADIAL AND AZIMUTH STATIONS ON CALCULATED ROTOR CHARACTERISTICS FOR A SAMPLE FLIGHT CONDITION

[$\mu = 0.50$; $\lambda = -0.091$; $\theta_0 = 15^\circ$; $\theta_1 = -8^\circ$; $\Omega R = 750$ ft/sec; $\gamma' = 1.0$; $\xi = 0$; $x_c = 0.15$]

(a) Part I solutions

Δx	$\Delta\psi$, deg	a_0, deg	a_1, deg	b_1, deg	a_2, deg	b_2, deg	a_3, deg	b_3, deg
0.1063	10	2.23	11.34	2.67	1.022	0.114	-0.001	0.186
	20	2.22	11.32	2.70	1.017	.117	-.001	.184
	45	2.14	11.16	2.83	.938	.179	.056	.144
.2125	10	2.18	10.91	2.63	1.021	.121	.004	.188
	20	2.18	10.96	2.65	1.017	.125	-.001	.186
	45	2.18	11.21	2.79	.959	.183	.058	.153

(b) Part II solutions[a]

Δx	$\Delta\psi$, deg	C_T/σ	$\Delta C_T/\sigma$, percent	$C_{P,o}/\sigma$	$\Delta C_{P,o}/\sigma$, percent	C_P/σ	$\Delta C_P/\sigma$, percent	a', deg	$\Delta a'$, deg	b', deg	$\Delta b'$, deg
0.1063	10	0.0685	----	0.2151	----	0.1761	---	13.75	----	-0.47	----
	20	.0697	1.8	.2130	1.0	.1785	1.4	13.99	0.24	-.32	0.15
	45	.0654	-4.5	.2278	5.9	.1842	4.6	14.39	.64	-1.08	-.61
.2125	10	.0673	-1.7	.2169	.8	.1745	-.9	13.69	-.06	-.52	-.07
	20	.0686	.1	.2151	0	.1775	.8	13.94	.19	-.36	.11
	45	.0660	-3.7	.2289	6.4	.1860	5.6	14.40	.65	-1.16	-.69
	45	.0672	-1.7	.2158	.3	.1765	.2	14.11	.36	.02	.49

[a]All part II solutions (except the last one) were calculated with flapping coefficients corresponding to $\Delta x = 0.2125$ and $\Delta\psi = 45^\circ$. The last solution was calculated with same values of a_0, a_1, and b_1, but with a_2, b_2, a_3, and b_3 assumed equal to zero.

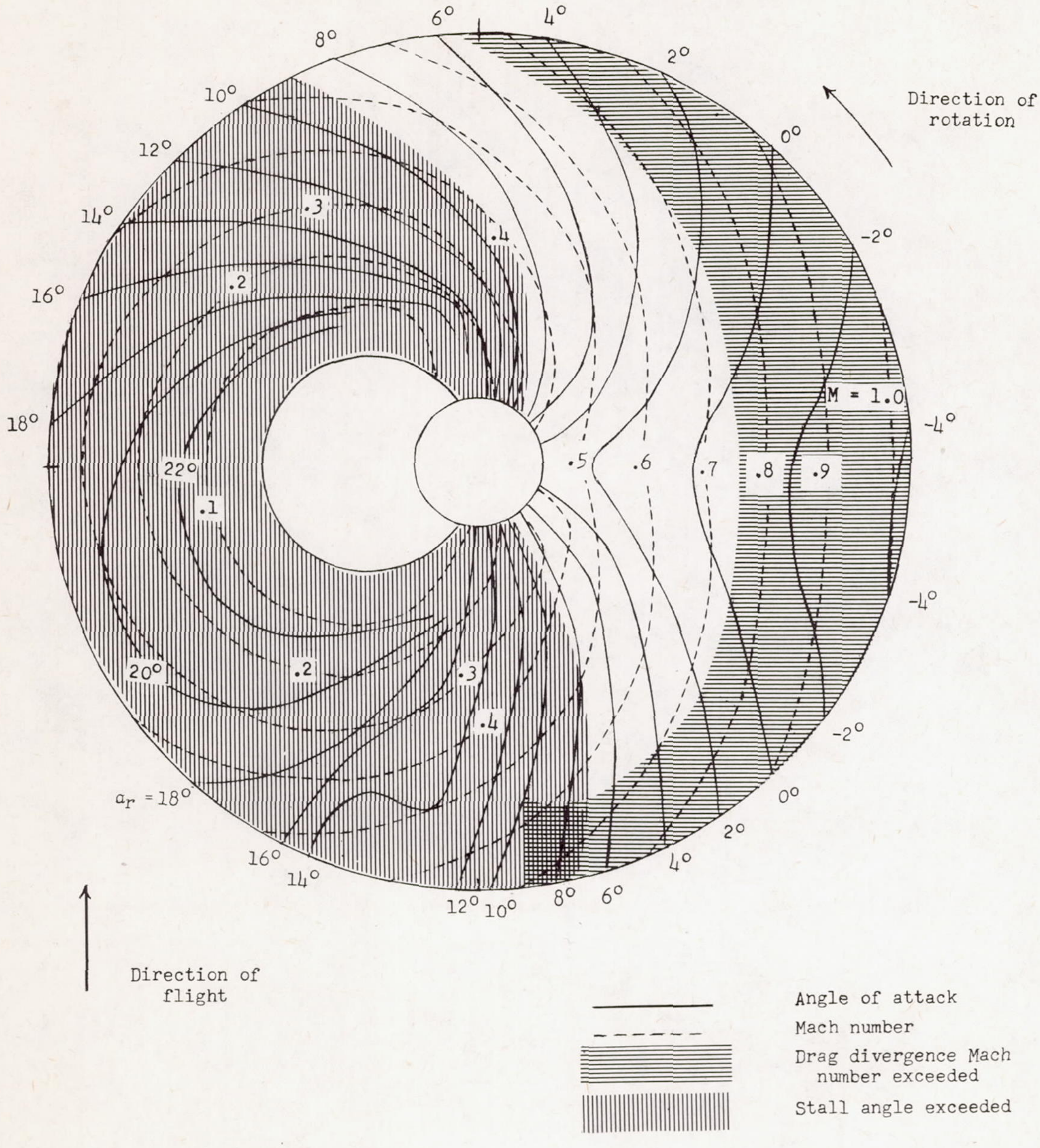

Figure 1.- Angle of attack and Mach number contours for sample flight condition. $\mu = 0.50$; $\lambda = -0.091$; $\theta_0 = 15^\circ$; $\theta_1 = -8^\circ$; $\Omega R = 750$ feet per second; $\gamma' = 1.0$; $\xi = 0$; $x_c = 0.15$.

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